THE ROLE OF LOGIC "COMMONLY SO CALLED" IN HEGEL'S SCIENCE OF LOGIC

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As any reader of Hegel's *Science of Logic* will quickly learn, for Hegel the term "logic" doesn't mean what it usually means—does not mean what Hegel refers to as "logic commonly so called [*gewöhnlich so genannten Logik*]" (Hegel, *Science of Logic*, 507; W, 6.243).[1] While many would agree with the claim that "the intuitive concept of consequence, the notion of one sentence following logically from others, is without doubt the most central concept in logic" (Etchemendy, *The Concept of Logical Consequence*, 6), commentators on Hegel's *Science of Logic* often take pains to distance this work from such considerations. David Gray Carlson writes that Hegel's *Science of Logic* is "radically *not* what Logic is for analytic philosophy – an exercise for clarifying mathematical or linguistic inferences" (Carlson, "Introduction", xi), and Frederick Beiser notes the "common misconception" that Hegel's dialectic "is some kind of alternative logic, having its own distinctive principles to compete with traditional logic". Hegel's dialectic, Beiser continues, was neither "meant to be a *formal* logic, one that determines the fundamental laws of inference governing all propositions, whatever their content", nor was it meant to "compete with formal logic". According to these and many similarly minded commentators, such confusion of Hegel's logic with logic commonly so-called follows from a failure to grasp what Hegel's logic *really* is. It really is, it is commonly said, metaphysics or ontology—an attempt "to determine the general structure of being" (Beiser, *Hegel*, 161).

And yet any approach to the *Science of Logic* will still need to address those early sections in Book 3 where Hegel certainly seems to engage with traditional logical issues such as conceptions of predication and logical consequence. But as

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[1]   References to English translations of Hegel's texts will be followed by volume and page references to *Werke in Zwanzig Bände*, designated "W".

Carlson points out, Book 3 of the Science of Logic, the so-called subjective logic, is that part of the text where "scholarly comment is rare indeed" ("Introduction", xi). Consistent with the attempt to distance Hegel from traditional logical concerns, some even deny that Hegel actually is here talking at all about judgements and inferences in the standard sense of these terms. Thus Andre Doz describes the "subjective logic" as simply a continuation of the "objective logic" of Books 1 and 2, in which the categorial structures discussed are understood as determinations of *being*. Thus Hegel's notions of "concept, judgement, etc." (with this list presumably meant to include "syllogism") in the subjective logic "are only [*ne sont que*] more developed forms or modes of being" like those in the earlier *objective* logic such as "becoming, finitude, infinite, quantity, number, essence, ground, substance, cause" (Doz, *La logique de Hegel*, 22). While this claim is quoted with apparent approval by Stephen Houlgate (*The Opening of Hegel's Logic*, 116), Doz's "*ne sont que*" sits awkwardly with Houlgate's insistence that "concept", "judgment", and "syllogism" refer to "structures of being, *as well as* categories of thought" (Ibid, emphasis added). But if we now align ourselves with Houlgate's reading, it would seem that the strategy of distancing Hegel from any direct treatment of logical consequence risks criticizing *him* in terms that he, in the *Lectures on the History of Philosophy*, uses of Aristotle with clear critical intent, when he compares Aristotle to the Stoics.

The Stoics, writes Hegel, "developed *formal logic* because they had made thinking their principle", and with them, logic was "no longer as with Aristotle, at least in regard to the categories, undecided as to whether the forms of the understanding are not at the same time the realities of things; for the forms of thought are set forth as such for themselves [*sind als solche für sich gesetzt*]" (*Lectures* II, 255; W, 19.274). Seemingly the Stoics had regarded *thoughts* as the objects of logical thought in something like the modern subjective sense, and had set forth for themselves the "forms" of these thoughts, *as thoughts*. In contrast, the suggestion seems to be, that at least when discussing the categories of thought, Aristotle had never made it clear as to whether these categories were primarily features *of thoughts themselves* or features of *that which such thoughts were about*, features of "being". While Hegel's speculative approach clearly aims at the identity of thought and being,

there are clearly unhelpful ways in which this identity can be asserted. The indeterminacy of Aristotle's approach here seems to be such a way.

Hegel's comments here cohere with what he elsewhere says about the aspirations and limitations of Aristotle's metaphysics. Aristotle must be understood as, like Plato, a speculative thinker: the chief moment of his philosophy is the idea that "the energy of thinking and the object of thought are the same" (*Lectures* II, 148; W, 19.163). Here Hegel directs us to Aristotle's discussion of divine thought in *Metaphysics*, Book XII. The perfect object of thought must be thought itself, and thinking is "thinking of thinking" (Aristotle, *Metaphysics* XII, IX, 4; Hegel, *Lectures* II, 151; 19.163). But this must be reconciled with the quasi-empiricist *manner* in which Aristotle presents his metaphysics such that the categories always appear within the everyday presentations of *perception* (*Lectures* II, 131; W, 19.146), with the suggestion that knowledge and perception are always of something *other* than thought. Aristotle's solution had been to claim that in the speculative sciences "the formula or the act of thinking is the object … since thought and the object of thought are not different in the case of things which contain no matter" (Aristotle, *Metaphysics,* XII, IX, 5; Hegel, *Lectures* II, 151; W, 19.166). Recall that for Aristotle, perception is the very taking of the form of the individual thing into the mind (*nous*), so that there, freed of the matter (that is merely the potential for being, and not being itself) the form can exist as both act of thinking and object of thought.

But we can appreciate the limitations that *this way* of conceiving of the identity of thought and being can pose for thinking about thought itself. Aristotle's approach here is at the heart of what Tyler Burge has labelled the "traditional view" of the nature of thought and its components (Burge, "Concepts, Definitions and Meanings", 291–4). According to Burge, thoughts on this classic account, which Burge himself traces to Aristotle, are taken to be "intrinsically representational" in that (1) their representational properties are used to explain the representational properties of the sentences used to express them, and (2) that thoughts differ from the derivatively representational sentences that express them in that while linguistic entities have properties *other than* representational ones, thought has *only representational properties*. Thoughts so conceived, we might say, are "transparent"

in having no features allowing them to be identified independently of the objects they are about. We can thus appreciate how, when thinking *about* thoughts such as in logical reflection upon judgments or inferences, we might never be clear as to whether it is the *thoughts themselves* that are actually being reflected upon, or whether reflection is simply about whatever those reflected upon thoughts are *themselves* about. That is, using Hegel's distinction, it would never be clear as to whether it was "the forms of the understanding" themselves that one had in mind, or the "realities of things" that those thoughts were purported to disclose. While Hegel's speculative thought demands the ultimate unity of thought and being, this unity must be one in which the *difference* of thought and being is nevertheless acknowledged, and Aristotle's approach to speculative unity seems to foreclose this possibility.

Hegel seems to suggest that by finding ways of conceiving *the thoughts themselves* as objects of reflection and thus breaking with the "traditional view" the Stoics were able to progress beyond the impasse presented by Aristotle's indeterminacy about the relation of thought and being and thus acknowledge the important moment of the distinction between thought and being needed for any properly speculative unity. Hegel alludes to the significant feature of Stoic logic that it is "in part a grammar and a rhetoric" (*Lectures II,* 257; W, 19.276)—that is, in contrast to the Aristotelians, who regarded logic as about *thought*, the Stoics regarded *linguistic expressions* as the primary objects of logical reflection, and in this sense had available a straight-forward way of regarding thoughts as objects of investigation. However, while Hegel clearly distinguished between a judgment and the statement or *Satz* which expressed it, he nevertheless held the external expression of thought to be a necessary condition for the development of reflective thought *about it*. His brief comments on the Stoics' logical advances are, despite his criticism of their having taken logic in a formal, and non-speculative direction, therefore consistent with this "expressivist" dimension of his own position. We might then appreciate how the Stoics, because of those features of their thought that allowed them to develop formal logic beyond the limits of Aristotle's own formalism, could be important for Hegel in the development of logical thought. In short, Hegel's brief comments on the Stoics suggest that issues of formal logic cannot be as readily sidelined as they so commonly are for understanding Hegel's own account of the identity of thought and being.

In order to develop this theme I will turn to the discussions of judgement and inference in Hegel's "Subjective Logic" and attempt to discern the general features of Hegel's attitude to such "formal" issues and suggest some of the consequences that his particular treatment of formal logic might have for the overall project of a genuine "science" of logic. Here I can only make a tentative start to such a project, and will review only parts of Hegel's treatment of the structures of judgement and inference (syllogism) in the subjective logic with a view to bringing into focus his diagnosis of a contradiction *within* existing approaches to these topics, a contradiction that in turn Hegel uses to drive his own account. In particular I will suggest that Hegel finds in the logical tradition two extant conceptions of predication (or two functions of the copula) and that this duality is carried over into his treatment of logical consequence. In brief, the relation of predication at the heart of judgement can be understood *either* as the containment or "inherence" (*Inhärenz*) of the predicate in the subject, *or as the* "subsumption" (*Subsumtion*) of the subject under the predicate. These conflicting interpretations of the predicative relation can, I suggest, then be traced through his treatment, which is at once both systematic and historical, of logical *consequence* itself. The dual conceptions of consequence that he discerns in formal logical thought are ones that are familiar in the history of logic. For some theorists, the consequence relation is fundamentally one that holds among premises and conclusion understood in terms of their mere truth or falsity. In short, to say q is a consequence of p is to say that p cannot be true and q false. For others, however, this fails to capture the sense in which in a valid inference the conclusion *follows from* the premises (Asmus and Restall, "A History of the Consequence Relations"). We might call these weak and strong conceptions of consequence respectively, and align them with the differences separating the logical thought of the Stoics from that of Aristotle,[2] but as we will see, these differences also appear internal to Aristotle's own approach to consequence.

In Hegel's sketch, the strong conception of consequence in the classical Aristotelian term-logical syllogism was characterized by the transitivity of the inherence relation found in the so-called "perfect" syllogisms. But perfect syllogisms

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[2]   For a modern version, consider the opposed approaches of Russell's "material implication" and C. I. Lewis's "strict implication".

are restricted to syllogisms in the "first figure", and Aristotle had to employ an *additional* conception of consequence in order to *reduce* the so-called imperfect syllogisms of the second and third figures to instances of some perfect syllogism in the first. But the "conversion rules" employed do not have the sort of intuitive immediacy found in the transitivity of inherence relations. Rather, they had to be understood, I will suggest, as appealing to the weaker sense of consequence, the sense captured merely by the relations holding between premises and conclusions considered in terms of their truth or falsity—an approach more consistent with the more proposition-based approach of the Stoics.[3]

It is this dual structure that underlies Hegel's treatment of the syllogisms in the subjective logic, from those in the traditional first figure through those in the second and third figures. In this process a greater reliance comes to be placed on subsumption rather than inherence relations. In the post-Aristotelian fourth-figure or "mathematical" syllogism, one witnesses the entire collapse of the features of judgement that had allowed both inherence *and* subsumption models to be established, and inferences in this figure can only be understood in terms of a new, *radicalized* version of the "subsumption model". With the breakdown of the conditions for the original intuitive understanding of the consequence relation (transitivity of inherence relations), a new conception of validity was required, and this was provided by Leibniz's radically new conception of judgement structure, a conception that allowed a *mathematizable* approach to patterns of inference based on the recently re-emerged (and non-Greek) science of algebra. But this was a development in which something "spiritual"—thought—had become reduced entirely to a mechanical process, eliminating the role of consciousness. The whole tradition of formal logic had thus in some sense ended in a type of *reductio ad absurdum*, calling for a revision of its original presuppositions. *Hegel's own* logic, I suggest, was meant to respond to this call, but neither its character nor its justification can be properly understood in isolation from his diagnosis of the problematic state into which logic

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[3] From his lucid comments on the development of ancient logic at the hands of the Stoics in his *Lectures on the History of Philosophy*, volume II, it is clear that Hegel was appreciative of the distinctiveness of Stoic logic from the Aristotelian syllogistic, and of the fact that Stoic logic was fundamentally a *propositional* rather than a *term* logic. (*Lectures*, II, 249–257; W, 19.268–277).

"commonly so called" had been led. If this is true, exactly what is at stake in calling Hegel's logic "metaphysical" or "ontological" cannot be brought into focus without considering the question of Hegel's engagement with "logic commonly so-called".

1. JUDGEMENT, THOUGHT AND LANGUAGE

On Hegel's view, a judgement (*Urteil*) involves a division (*Teilung*) into parts (*Teile*) (Hegel, *Science of Logic*, 552; W, 6.304). These parts are traditionally thought of as subject and predicate, but we must be careful using such grammatical terms as names for the parts of the judgement itself, as Hegel insists that the parts of the *judgement* are determinations of "the concept [*der Begriff*]". Here too, we must be careful, as a "concept" is usually thought of as expressed by the predicate term and said *of* the subject, which is typically designated by a *name*, which may be the name of an individual (a proper name) or of a kind (a common name). As we will see, Hegel has a way of eliminating *names* from judgements, and with this, the concept of "concept" that pairs concepts with names must change too. Hegel's peculiar concept of *concept* should be thought of as, at least in the first instance, closer to *Kant's* "I think", the representation that can accompany all other representations, thereby allowing for the rational *unity* of all those representations: the *transcendental unity of apperception* (*Science of Logic*, 515; W, 6.254). Thus in the first instance, we might think of *judging* as an act in which *a judging subject*, an I, comes to have a determinate form—that is, comes to have some determinate cognitive commitment, such as a belief or intention to act. The judgement, says Hegel, is the "first realization" of the concept and its "entry into *existence* as *determinate being*" (*Science of Logic*, 550; W, 6.302).

In the *Phenomenology of Spirit* Hegel refers to *language* as providing the medium for the determinate existence [*Dasein*] of spirit (*Phenomenology of Spirit*, §652; W, 3.478), and so we might think of the division of the concept as involving a process in which *physically discriminable linguistic components*—the temporally distinct parts of speech, or spatially discriminable elements of writing—come to express something mental, such as a thought. But here again one must tread carefully: Hegel, I suggest, tries to avoid either of two contrarily opposed common conceptions

of the relation of thought to language. On the one hand, with the idea that thought *requires* external language to become determinate, Hegel is clearly opposed to any form of Cartesian subjectivism in which the mind can be thought of as a type of isolable internal realm with determinate thought contents of which it is immediately aware. At the same time, however, in refusing to *identify* the logical structure of judgement with the grammatical structure of its linguistic expression, Hegel seems equally opposed to what we might call linguistic *positivism*—the tendency to *reduce* thought to its linguistic expression as had been found in earlier thinkers such as J. G. Hamann, reducing logical to merely grammatical form ("Metacritique", 58).

Hegel's distinctive point here is that while we might initially take the linguistic expression of the judgement to be made up of subject and predicate functioning as names and relating to each other as merely grammatical or syntactic entities, the *judgement itself*—one might say the thought expressed by the utterance—is made up of *determinations of the concept*, and as such is not reducible to subject and predicate as merely grammatical determinations (*Science of Logic*, 550–1; W, 6.302–3).[4] "The subject and predicate" he says, "are names at first that receive their actual determination only as the judgement runs its course" (*Science of Logic*, 557; W, 6.311). In "running its course" we will see that rather than functioning as names which *fix* the meaning of the expression, the parts of the judgement must be considered as conceptual determinations that essentially allow for the process of *redetermination*.

Hegel describes the determinations into which the concept divides as singularity, particularity and universality.[5] To become determinate "the concept" has to divide into such determinations that are expressed in the subject and predicate

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[4]   In *this* sense, Hegel too exemplifies Burge's "traditional view": conceptual determinations as the elements of thought cannot be identified with those syntactically manipulable bits of language in which they are expressed. But in insisting on the necessity that thought be objectified in language in order to become self-conscious, Hegel departs from the spirit in which the "traditional view" had been *traditionally* understood.

[5]   Kant had spoken of the singular, particular and universal *uses* of concepts (*Lectures on Logic*, 589). While Hegel is critical of the idea of concepts as useable "things", the idea that ties the determination of the concepts to cognitive acts is clearly analogous.

places of the expression such that the logical forms of those expressions will be given by formulae such as "the singular is universal" or "the particular is universal" or "the singular is particular", and so on. And yet the *copulas* in these judgements seem to express the idea that these distinct and separated determinations of the concept are, in fact, *identical*, as the singular is said *to be* the universal, and so on. As the subject term of a judgement is typically taken as referring to the worldly item the judgement is about, and the predicate as expressing what is *thought about it*, the identity of subject and predicate expressed by the copula is itself an expression of the speculative identity of being and thought. It is these two contradictory aspects of the judgement—the *need* for "the concept" to separate into its different determinations in order to achieve determinacy (by being realized in the linguistic expression), and yet the stated underlying unity of these determinations *as* determinations of "the concept"—that will drive a dialectic allowing the two conceptual components of the judgement expressed in the subject and predicate terms to become successively *redetermined* as to their logical form. One consequence of this is that a single linguistic expression will be capable of having *different* conceptual, that is, logical forms.

2. A TAXONOMY OF LOGICAL FORMS OF JUDGEMENT

At the heart of Hegel's account of judgement is a distinction between two judgement forms with logically different structures. Hegel calls these judgements of existence (*Dasein*) and judgements of reflection, but he also distinguishes them as "qualitative" and "quantitative" judgements and as "judgements of inherence" and "judgements of subsumption" (*Science of Logic*, 557 & 570; W, 6.311 & 328). The last of these distinctions is made on the basis of two different ways in which the copula or predication is understood: in judgements of inherence (or existence), the predicate is thought to *inhere in* the subject, while in judgements of subsumption (or reflection) the subject is thought of as *subsumed under* the predicate. From here on I will use the inherence–subsumption distinction to characterize these two logical forms as this thematizes the issues that I want to trace through the later treatment of inference.

Hegel treats the initial most *immediate* judgement form as one in which an abstractly singular subject term is united with an equally abstract universal. "[T]he subject as such", Hegel says, "is at first only a kind of *name*; *what it is*, is first

enunciated only by the predicate which contains *being* in the sense of the concept" (*Science of Logic*, 551; W, 6.303). Hegel portrays such types of abstract oppositions as problems to be solved, and the means for their solution is an understanding of how they can be "mediated". In this case, the problematically opposed name-like abstractly singular subject term and the abstract general concept suggest Kant's intuition–concept distinction, and it is obvious that there are parallels. For Kant, the genus "representation" divides into the species of concept and intuition in a way that it is only the combination of elements coming from each of these two species that allows our judgements to attain a determinate content: "thoughts without content are empty, intuitions without concepts are blind" (*Critique of Pure Reason*, A51/B76). But the stark heterogeneity of these two types of representations for Kant had made the question of this "synthesis" difficult, if not impossible, to answer. Part of Hegel's answer involves treating what Kant thought of as a *non-conceptual* representation, intuition, as conceptual, or, more accurately, as *the concept* in one of its determinate forms—that of singularity—and as playing a role in a judgement by being copulated with another, different, conceptual determination.

Kantian intuitions, qua singular representations, seem *name-like*, but naming is not going to be a *foundational* notion for Hegel's approach to semantics, although it plays a necessary role. Understood as a proposition or *Satz*, by which Hegel means an objective concatenation of word-objects considered in isolation from any "judgement" being expressed by them, an utterance is made up of subject and predicate names that are linked in spatio-temporal relations to worldly elements: "Of course, *grammatically* speaking this kind of subjective relation that proceeds from the indifferent externality of subject and predicate is perfectly valid, for it is *words* that are here externally combined. – It can also be mentioned in this context that a *proposition* can indeed have a subject and predicate in a grammatical sense without however being a *judgement* for that" (Hegel, *Science of Logic,* pp. 552–3; W, 6.305). But when the subject term is grasped properly as *the concept in its determination as singularity* in the context of a judgement that says, for example, that "the singular" is "the universal" and so identifies subject and predicate, its meaning is no longer understood as *fixed* in the way that it would be if considered as a name. It will be crucial for Hegel that the meaning of both subject and predicate terms of an expressed

judgement can be *redetermined* in a way that the treatment of the subject term *as a name* would make impossible.

While Aristotle's adherence to the "traditional view" may have obscured the nature of the objects of logical reflection, in the *Prior Analytics* he did, of course, attempt to give thought a determinate thinkable form and so initiated "formal" logic. Thus, in order to abstract away from particular worldly contents, Aristotle had employed Greek letters such as *alpha*, *beta* and *gamma* to designate subject and predicate terms. In contrast, in discussing syllogisms, Hegel will use the names of conceptual determinations occupying subject and predicate places to yield forms such as "the singular is the universal", and to this extent at least, Hegel too employs a type of formalism. But the logical-form-expressing claim, "the singular is the universal" can, we are told, now be taken in two distinct ways. It can be taken as saying that "the singular is universal" *or* it can be equally taken as saying "*the universal is singular*" (Hegel, *Science of Logic*, 559; W, 6.314). While the former formula is said to express the judgement according to its *form* [*Form*], the latter, expressing the situation in the reverse way, is said to express that judgement according to its *content* [*Inhalt*] (*Science of Logic*, 560–1; W, 6.315). With this Hegel seems to be seeking ways to capture the different *functions* allowed by Kant's intuition–concept distinction. Intuitions paradigmatically serve the function of providing content for cognition while concepts determine the form of that content, allowing the judgement to be integrated into the transcendental unity of apperception. But rather than fixing these functions in two formally heterogeneous representations, Hegel will assign them to two different logical forms, as the pair of inverted formulae provides the basis for the distinction between the two basic judgement types: content-providing judgements of inherence and form-determining judgements of subsumption respectively.

The logical structure of the judgement of inherence (to be read as "the universal (predicate) is (like the subject) singular") is such that its otherwise universal predicate is to be construed *as a singular*. Such a "singularized universal" seems close to what is described in contemporary metaphysics as "tropes", "abstract particulars" or "property instances" (see, for example, Bacon, *Universals and Property Instances*). On this way of thinking, to attribute a certain property to a particular thing is to

attribute to it the property that *it and it alone* has: to predicate "red" of some particular rose is to attribute to it the particular redness that *it* has and not the general property redness that it has in common with, say, post-boxes and fire-engines. Hegel's reasons for treating the otherwise universal predicate as a *singular* become plain when it is understood that a judgement of inherence specifies the logical form taken by content conferring *perceptual* judgements. The immediate way to take this is to regard the predicate as functioning as a "kind of name" for this singularized universal—the thing's perceivable inherent property.[6] In contrast, in the judgement of subsumption, the abstract universality of the *predicate* is conveyed to the subject, resulting in judgements tailored to the expression of *form*. In this type of judgement, which allows explicit quantification (Hegel, *Science of Logic,* 568–75; W, 6.326–35), the universal predicate is not going to be understood in a concrete trope-like in this way, but as "abstract". If I say "all the furniture in Alfred's room is red", the predicate will be understood as an "abstract universal", and the relation of the subject to the predicate will be that of subsumption rather than inherence.[7]

An important feature of the judgement of inherence that becomes explicit in Hegel's discussion of its *negative* form is that a sortal or kind term is found in subject position. That is, just as its predicate term takes on characteristics of a singular, so too does its subject term take on the characteristics of a universal.[8] While the most

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[6]   Of course in appealing to tropes to illuminate Hegel's singularized universals, I am not suggesting that Hegel is committed to anything like an ontology of tropes in the manner of contemporary trope theorists.

[7]   It is precisely this feature of universally quantified affirmative judgements that led Russell, for example, to treat the logical form involved as that of a *conditional* ("On Denoting"). Here, "All the furniture in Alfred's room is red" becoming something like "For everything in Alfred's room, if it is a piece of furniture, then it is red".

[8]   Hegel's argument runs broadly along the lines that the negation of a judgement such as "the rose is red" still carries a certain positive content, namely, that the rose has some *other* colour. "When it is said that, for instance, the rose is *not* red, only the *determinateness* of the predicate is thereby denied and thus separated from the universality which equally attaches to it; the universal sphere, *color*, is retained; if the rose is not red, it is nonetheless assumed that it has a color, though another color. From the side of this universal sphere, the judgement is still positive." (Hegel, *Science of Logic*, 565; W, 6.322). This implies something about the nature of the subject of the judgement—that it is the *kind* of thing that can be coloured in particular ways. Roses, for example, can be variously coloured, but numbers cannot.

immediate judgement form might have had a name-like demonstrative "this" in its subject term, the judgement of inherence will be a judgement about a "this such"—considered as a name, it would name a *particular instance of a kind*. In this way, the *subject* of a judgement of inherence can be seen to align with what in the *Phenomenology of Spirit* Hegel treats as the object of "Perception"—a "this such" rather than a bare "this" (*Phenomenology of Spirit*, Ch. 2).

In contrast, however, in the judgement of subsumption, the subsumed subject term will, at least in the first instance, be determined as a "singularity as such"—that is, it is conceived as singular or "bare particular" *without* determination by some "kind" concept. Thus, while the logical form of judgements of inherence reflects a type of Aristotelian, term-logical provenance, we might say that judgements of reflection reflect something more like a propositional-logical one, as found in the Stoics.[9] And it should be remembered that the functional task for this form of judgement is no longer that of content acquisition as what comes to the fore here is the requirement of integration of the judgement into the unified web of belief. Analogously, neither will the predicate of the judgement of reflection be understood as a singularized universal—trope or property instance. Rather it will be, Hegel says, an "*essential universal*", or, we might say, a "universal as such", the task of which is to *subsume* the subject term. Here, it is, Hegel says, "the *universal* or the predicate" which "constitutes *the basis* against which the subject is to be measured and determined accordingly" (*Science of Logic*, 569; W, 6.327). Here the bare singular subject term is treated *as* a universal in the sense that it is *universalizable*, something that along with other things will "satisfy" that universal or count as a member of a class that is characterized by the universal.

To sum up: in the context of acts of perception, a judgement will be thought of as having a logical form in which a trope-like singularized universal inheres in a subject treated as an instance of a kind. In contrast, in contexts where we are integrating the content of a judgement into a system of beliefs, we will think of the

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[9] The Stoics are now considered as the inventors of propositional logic. Thus in contrast to Aristotle's term logic, the Stoics favoured *singular judgements* and "external" rather than term negation (Bobzien, "Logic").

judgement form as one in which a singular subject, free of any determination as member of a kind, is subsumed under an abstractly universal predicate that holds of other things as well.

In this basic divide between the two opposed judgement forms, I suggest we can see Hegel as drawing on doctrines from the formal logic of Gottfried Ploucquet, whose Leibniz-inspired logic was taught at the Tübingen *Stift* during Hegel's years there.[10] Ploucquet had differentiated between two different senses of "particularity" characterizing the "particular judgements" of the Aristotle's syllogistic, that he called "exclusive" and "comprehensive" particularity respectively (Ploucquet, *Logik*, §§12–16)—a distinction that seems to align with the dual functions of *content acquisition* and *logical integration* that underlies Hegel's separation of judgement types. Thus, in the "exclusive" use of the particular subject term, "some A" or "some As", to say that *some As* are B implies that *not all* As are B.[11] Here Ploucquet seems to regard the subject term as effectively functioning as a type of name, picking out those *particular* instances of the kind A that the speaker has in mind, with the implicit denial that *all* As are B. On this interpretation, a sentence like "Some philosophers are bearded" would be read as "Certain philosophers are bearded" and have a logical form akin to a conjunction, such as "Plato is bearded", "Aristotle is bearded", "Socrates is bearded", and so on, as in the traditional nominalist interpretation of quantified judgements. Other uses of "some" however, and in particular, specifically all strictly *logical* uses, do not have this implication. For example, Aristotle's rules of conversion specify that "all philosophers are bearded" implies "some philosophers are bearded", a relation which permits *both claims* to be true, which is not possible on the "exclusive" reading. Here, the truth of the particular judgement cannot exclude that of the

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[10]   While Ploucquet had retired by the time Hegel was a student at Tübingen, he was still active during Hegel's first years there, and his approach to logic is regarded as having been canonical at that institution (Frank, *Auswege aus dem Deutsche Idealismus*, ch. 13).

[11]   Franz ("Einleitung, xlvi–xlvii) notes Hegel's use of Ploucquet's distinction, but seems to imply that Hegel in simply inconsistent in his treatment of particularity as exclusive or comprehensive. But rather than inconsistent, the difference in Hegel goes along with the difference between judgements of inherence and judgements of subsumption.

universal judgement. In such uses, "some" is clearly used *without* any particular individuals in mind.

For Hegel, this idea of the semantic ambiguity of what is expressed as a single linguistic item, such as "some philosophers", is a mark of the essential semantic *re-determinability* of our expressed claims, and is central to his "idealist" approach to logical form. Ploucquet's "exclusive" use would correspond to the quasi-naming role of the subject term in its immediacy, but as the properly logical form of the expressed judgement gets redetermined in different functional contexts it gets the properties of Ploucquet's "comprehensive" sense of particularity.

Hegel now passes on to judgements of other logical forms, judgements of necessity (categorical judgements; hypothetical judgements and disjunctive judgements) and "judgements of the concept" (assertoric judgements; problematic judgements and apodictic judgements). The most developed of the judgements of the concept, apodictic judgements, are shown to be implicit syllogisms. While there is, of course, much that is relevant in these discussions, for our purposes we can transit into his treatment of inference on the basis of what has been learnt from judgements of inherence and subsumption only.

3. INFERENCE

We can now follow this distinction between these two understandings of predication into Hegel's treatment of *inference* in his treatment of the "syllogisms". While this is first and foremost a *systematic* development of the syllogistic form, we can recognize in it a reflection on the historical fate of the Aristotelian syllogism in which the idea of logical consequence had been based on the interpretation of predication as a relation of *inherence*. The relation of consequence based on the transitivity of inherence will be seen to fail to be comprehensive *even in Aristotle's own system*, and to be in need of a conception of consequence based on the alternate conception of predication as *subsumption*. Indeed, the final outcome of this for Hegel will be a type of *reductio* in which a characteristic at the centre of *thought*, the notion of consequence, is reduced to a mere lifeless mechanism.

In his treatment of the first-figure syllogism, Hegel suggests that Aristotle confined himself "to the mere relation of *inherence* by defining the nature of the syllogism as follows: *When three terms are so related to each other that the one extreme is in the entire middle term, and this middle term is in the entire other extreme, then these two extremes are necessarily united in the conclusion*. What is here expressed is the repetition of the *equal relation* of inherence of the one extreme to the middle term, and then again of this last to the other extreme, rather than the determinateness of the three terms to each other" (*Science of Logic*, 591; W, 6.356).[12] But this applies *only* to the so-called first-figure syllogism, and in Aristotle's second and third figures, in which it is no longer the case that the "middle term" plays the role of predicate in one premise and subject in the other, this intuitively plausible model of consequence based on the idea of the transitivity of the containment or inherence relation no longer holds. Thus the "imperfect" syllogisms in the second and third figures require *proof*, which for Aristotle is to be effected by "conversion" rules that allow such transformations of the sentences expressing the judgments as the reversal of subject and predicate terms in one or other of the premises so that an intuitively valid first-figure syllogism is produced.

Hegel interprets this need to prove the *imperfect* moods by reduction to some perfect mood in the first figure as indicating the *dependence* of the intuitive basis of the first figure *on* such *non*-inherence conceptions of consequence as found in the conversion rules. Thus he says in the context of the second-figure syllogism:

> In so far as it is regarded as only a subjective syllogism that runs its course in external reflection, we can then take it as a *species* of syllogistic inference that should conform to the genus, namely the general schema S-P-U. But it does not at the moment conform to it; its two premises are P-S or S-P and S-U; the middle term is in both cases the one which is subsumed or is the subject in which the two other terms thus inhere – is not therefore a middle term that in one case would subsume or be predicate, and in the other would be subsumed

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[12]   The primacy if inherence is consistent with Aristotle's taking the empirical object as the embodiment of logical form.

or be subject, or a middle in which one of the terms would inhere but would itself inhere in the other. – The true meaning of this syllogism's lack of conformity to the general form of the syllogism is that the latter has passed over into it, for its truth consists in being a subjective, contingent conjoining of terms. (*Science of Logic*, 599; W, 6.367)

Here Hegel treats the transitivity that secures inference in perfect figures as interpretable either as between inherence relations *or* subsumptive ones, despite his claim that Aristotle had relied on the transitivity of inherence. In the eighteenth century these two readings were employed and commonly confused, as can be seen in Euler's account of the "principles of the containing and contained" in relation to his logic diagrams.[13] But Hegel's principle point is clear: the immediate conception of inference as found in the "perfect" syllogisms is not as *perfect* as it appeared to be. While the first figure is based upon the transitivity of a relation that can be understood either as inherence or subsumption, the conversion rules rely on a weak conception of consequence that fits more with the interpretation of predication as subsumption *rather than* inherence. The development of the extensional interpretation of concepts as part of the general development of the subsumption model becomes focused upon by Hegel in his account of the so-called "fourth figure" syllogism in which the intuitive understanding of consequence relations based on the idea of conceptual containment entirely vanishes, demanding a *new* conception of conceptual relations that is supplied by Leibniz.

4. THE FOURTH-FIGURE OR "MATHEMATICAL" SYLLOGISM

This post-Aristotelian fourth-figure syllogism confounds the understanding of inference on the model of the transitivity of containment or inherence. In syllogisms in the fourth figure, the major term, which is a *predicate* in the major premise,

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[13]   In his *Letters to a German Princess*, when explaining his logic diagrams Euler appeals to the "principles of the containing and contained" as the those upon which all inference is based (Letter CIV), but seems to mix "intensional" and "extension" readings. Thus he describes how a general notion "contains [renferme] an infinite number of individual objects" (Letter CII). The idea of the "extension" of a concept seems to originate with the *Port Royal* logic, but there, as well as in subsequent developments in the 18th century, there appears to be considerable conflation between notions now distinguished as "intension" and "extension".

becomes a *subject* of the conclusion, while the minor term, which had been the subject of the minor premise, becomes a *predicate* of the conclusion. Hegel describes the conception of predication operating in this syllogism in the following way:

> *if two things or two determinations are equal to a third, then they are equal to each other*. – The relation of inherence or subsumption of terms is done away with.
>
> A 'third' is in general the mediating term; but this third has absolutely no determination as against the extremes. (*Science of Logic*, 602; W, 6.371).

Hegel goes on to discuss the algebraicized logic or "logical calculus" of Leibniz and Ploucquet in this context, and it is clear that the underlying idea behind the mathematical syllogism is Leibniz's radically *nominalist* reinterpretation of the nature of predication. Rather than thinking of the judgement in the traditional sense as the joining of a *universal-naming* predicate to a *single substance-naming* subject, Leibniz, adopting an approach from Johannes Raue (Angelelli, "On Johannes Raue's Logic"), had treated the subject term as *itself* a predicate, such that the judgement "S is P" was to be read as *identifying* terms S and P in the sense of each term's *being true of* some "third", a "*tertium commune*", *not named* in the judgement, and so, as Hegel says, having "absolutely no determination as against the extremes" (*Science of Logic*, 602; W, 6.371). We can appreciate how this understanding of the judgement had been arrived at by an *extension* of the idea of the subsumption of the subject under the predicate making up the judgement of subsumption. But now, rather than it being the *subject* term that is subsumed under the predicate term, the expressed subject term is *itself* treated as a predicate, allowing the posited unnamed "third" to be subsumed under *both* subject *and* predicate terms. Thus, as Hegel points out, this allows all the terms set out in the syllogism to be considered as universals, the fourth-figure syllogism having the logical structure U–U–U. The mediating determination of "particularity" has been eliminated, and "singularity" features as the bare singularity of those "thirds" subsumed under the various abstract universals making up the judgement structure. We are, it would seem, back to the *very first* conception of judgement form of abstractly universal predicates being predicated of abstractly singular subjects, with the difference now being that these judgements appear only after the overt logical analyses of the expressed judgements.

We can think of this move as an extension of the "quantitative" features of the "subsumption" structure of reflective judgements and inferences. Predicate terms on this reading now become the names of *classes*, and judgements express the relations between those classes designated by the subject and predicate *predicates* respectively. It is this that allowed the treatment of logic as a *calculus*. This radically *non-Aristotelian* analysis of judgement form was known and debated within German scientific circles in the second half of the eighteenth century (Capozzi and Roncaglia, "Logic and the Philosophy of Logic"). Ploucquet seems to have been the last and possibly most advanced figure in this algebraic tradition.

Hegel says of the mathematical syllogism that it "ranks in mathematics as an *axiom*, *as a first self-explanatory proposition* which is neither capable nor in need of proof, i.e., of any mediation – which neither presupposes anything else nor can be derived from anything else" (*Science of Logic*, 602; W, 6.372). But it would seem that the *axioms* of mathematical syllogisms do *not* in fact have the type of immediate intuitable "certainty" that had characterized the "inherence" construal of the copula found in the perfect *traditional* syllogisms. Thus, Hegel notes that "if we take a closer look at this prerogative that the proposition claims, of being immediately *self-evident*, we find that it lies in *its formalism*, in the fact that it abstracts from every qualitative diversity of determinations and admits their quantitative equality or inequality" (*Science of Logic*, 602–3; W, 6.372, second emphasis added). The mathematical syllogism is the "*merely negative result*" of the process of the development of the formal syllogism and as such has amounted to "the dissolution of the qualitative determinations of form into the merely quantitative, mathematical syllogism" (*Science of Logic*, 604; W, 6.373). But the components of the mathematical syllogism for Hegel no longer express *judgements*, rather, they are, rather, merely "propositions" (*Sätze*)—that is, concatenations of names considered as mere *things* that can be linked in regular patterns with other *natural,* in the sense of spatio-temporal, entities. Thus, "the concept and its determinations" do not enter into this type of syllogism *at all*, and "the understanding is also not faced here by even the formal, abstract determinations of the concept" (*Science of Logic*, 603; W, 6.372). We are, in effect, now in the realm

of *nature* in the determination of *externality* (Hegel, *Philosophy of Nature*, §§247–8).[14]

So, the elements of the mathematical syllogism, rather than expressing conceptual determinations, are now taken to be simple spatio-temporal entities to be manipulated according to explicit rules, and with this, thought itself becomes reduced to a type of mechanical "calculation" or, as would be said later, "computation". In short, the elements of this "notation" are no longer taken as entities existing *for* "spiritual" beings with consciousness who can recognize them as expressing an inner conceptual content. Rather, the formalization appeals to a type of "deployment" of the elements that is *essentially* mechanical, such that the mechanism itself comes to replace the original conscious subject *for* whom the formal syllogism had originally existed. Leibniz had only been able to imagine the development of machines capable of this, although he clearly saw the possibility of a mechanical *reduction* of certain activities of the mind. The actual implementation of the idea of the mechanization of thought had to wait until the twentieth century, but the idea of the *mechanization of logic* is explicit in the passage from Ploucquet that Hegel quotes with barely concealed horror. "I can teach the whole logic mechanically even to the uneducated, just as children are taught arithmetic, so that, if there is no error of calculation, it would be possible not to be tormented by the fear of erring in reasoning or of being deceived by falsities" (*Science of Logic*, 608, fn. 92; W, 6.379–80).[15] Ploucquet and Hegel were aware, of course, that arithmetical calculation could be "performed" by machines. Pascal had designed a machine capable of addition and subtraction, and Leibniz himself had designed one capable of carrying out multiplication and division. If logic could be successfully reduced to a calculus, it could be taught "mechanically" because it would, like addition and multiplication, be capable of being "performed"

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[14]   In this sense, Hegel's *Satz* might be seen to resemble Wittgenstein's idea of a *Satz* qua "fact" [*Tatsache*] regarded as a "picture" [*Bild*] of other facts—that is, as a model the elements of which are "combined with one another in a definite way" allowing it to represent [*vorstellen*] that "things are so combined with one another". Ludwig Wittgenstein, *Tractatus Logico-Philosophicus*, translated by C. K. Ogden (London: Routledge and Kegan Paul, 1922), 2.1–2.15.

[15]   Ploucquet is discussed in the context of the history of the mechanization of reasoning by logicians Marciszewski and Murawski (*Mechanization of Reasoning in a Historical Perspective*, Ch 4.2.3).

by machines. Such an idea, Hegel adds, "is surely the worst that can be said of an invention that bears on the presentation [*Darstellung*] of the science of logic" (*Science of Logic*, 608; W, 6.380).

A superficial reading of these passages may indeed reinforce the idea that Hegel is simply denouncing formal logic—a way of thinking about logic started by the Stoics which, in contrast with Aristotle's essentially "speculative" approach, treats logic as expressing "the activity of the understanding [*Tätigkeiten des Verstandes*]" (*Lectures on the History of Philosophy*, II, 255; W, 19.273). But as John Burbidge has stressed in relation to Hegel's logic, ("Where is the Place of the Understanding?") reason is not antithetical to the understanding: rather, reason is a type of thought which relies on the determination-fixing capacities *of* the understanding—a fixing of determinations that allows contradiction to be generated from within the operations of the understanding itself. Moreover, the idea that thought has to be brought to the point of being confronted by its death before grasping the means for a proper resolution of the underlying problem is surely a recurring theme in Hegel from the *Phenomenology*'s "struggle for recognition" (*Phenomenology of Spirit*, ch. 4) onwards. As Henry Harris has stressed (*Hegel's Ladder,* vol. 1, 585), "Observing Reason" in the *Phenomenology* must pass through the pseudo-science of *phrenology*, where it has been forced to the absurd identification of spirit with "a bone" (Hegel, *Phenomenology of Spirit*, ¶ 343; W, 3.255). Harris's description of phrenology as "the Calvary where singular Reason is crucified, and the spirit of 'absolute knowledge' rises from the grave" (*Hegel's Ladder*, vol. 1, 585) reminds us of the significance of *God's* very death in Hegel's interpretation of Christianity. Similarly, I suggest, Leibniz's logic represents the "ossification" of thought as it reduces the life of thought to the operations of a dead mechanism—here too, thought must somehow "rise from the grave", and cannot do so without *passing through* this process. The path to this can only be via the discovery of reason within the merely worldly *contents* that have been left with the elimination of the "external" consciousness that had been implicit in formal logic at its birth.

5. CONCLUSION

Let us return to the question of the nature of Hegel's "logic". As we have seen, for some readers, such as Andre Doz, Hegel's science of logic should be read simply and primarily as an "ontology", presumably able to be understood in isolation from any *specifically* logical doctrines as those dealing with the nature of judgment and inference. For others, such as Stephen Houlgate, Hegel's logic is meant to articulate structures that are *equally* structures of being and thought. But attention to Hegel's treatment of the limitations shackling Aristotle's speculative metaphysics as well as to limitations stemming from Aristotle's related understanding of predication as "inherence" suggest problems for any approach to thought and being as having an immediate unity understood in this way. Hegel engages with a greater range of traditional logical issues, and does so with a degree of sophistication than may be expected, and in a short space manages to sketch the basic contours of the history of formal logic from Aristotle and the Stoics to Leibniz and Ploucquet. In this sketch, Aristotle's attempt to make the structure of reason explicit by laying out the form of the syllogisms regarded as reflecting the structures of being is seen as relying on two different ways of understanding the nature of predication. Thus, in addition to the "traditional view" attributed to Aristotle by Burge, there is an internal "Stoic" dimension to Aristotle's logic, and so, the path to mechanical thought initiated by Stoic formalism cannot be regarded as representing some avoidable wrong turn taken by thinking that led to the modern "subjective" approach to logic. Rather, it grows out of the conditions for the very project of logic conceived of as thought thinking itself that is a direct expression of Greek speculative thought.

Nevertheless, in Hegel's account, "subjective" logic does eventually pass over into "objectivity", and some dimension of an "ontological reading" indisputably returns to logic. Thus the final form of the syllogism—the syllogism of necessity—we are told, "has attained the correspondence of its concept (or the middle term) and its existence (or the difference of the extremes). It has attained its truth – and with that it has stepped forth out of subjectivity into *objectivity*" (Hegel, *Science of Logic*, 590; W, 6.354). But we already had a premonition of this in the fate of the subjective syllogism itself, as Leibniz's radical reformulation of the nature of both predication and consequence had given rise to a conception of logic as a calculus in which

thought itself gets reduced to a type of *mere being*, that is, as merely mechanically manipulable bits of nature. As with Aristotle's initial starting point, this represents a *type of* "unity" of thought and being, but it is clearly a very different conception of such a unity from that envisaged by Aristotle.

Hegel portrays Leibniz's calculus with its intimations of the complete mechanization of thought as the outcome of a dialectic that develops as a necessary consequence of Aristotle's own starting point, and so Hegel's own retrieval of "speculative" thought, I suggest, must surely be understood in the light of this dialectic. There can be no simple return to Aristotle, and in order to understand how "logic" and "ontology" are meant to cohere for Hegel we cannot avoid the tasks of understanding his *Science of Logic* also *as* a logical doctrine in the more traditional sense of the word. It is certainly the case that Hegel's *science* of logic must be a properly *conceptual* thinking about conceptual thought itself, and as such cannot be reduced to or equated with logic "commonly so-called"—any formal logic which attempts to represent thought in forms that are not, in Hegel's terms, entirely conceptual, such as the forms of mathematics. It is also true that in some yet-to-be-specified sense of "metaphysics" or "ontology", Hegel's science of logic is best understood as constituting a metaphysics or ontology as is so often claimed. Nevertheless, it cannot, I suggest, be simply assumed from the outset how such a science of logic *as* ontology is meant to be understood, and nor can this be used to dismiss the relevance of Hegel's engagement with the tradition of formal logic.[16]

BIBLIOGRAPHY

Aner, K. *Gottfried Ploucquets Leben und Lehren*. Halle: Verlag von Max Niemeyer, 1909; republished, Hildesheim: Georg Olms Verlag, 1999.

Angelelli, Ignacio. "On Johannes Raue's Logic". In *Leibniz' Auseinandersetzung mit Vorgängern und Zeitgenossen*, edited by Ingrid Marchelwitz and Albert Heinekamp, 184–190. Stuttgart: Franz SteinVerlag, 1990.

Asmus, Conrad and Greg Restall. "A History of the Consequence Relations". In *Handbook of the History of Logic*. *Volume 11*. *Logic: a History of its Central Concepts*, edited by Dov M. Gabbay, Francis Jeffry Peletier and John Woods, 11–61. Amsterdam: Elsevier, 2012.

Bacon, John. *Universals and Property Instances—The Alphabet of Being*. Oxford: Blackwell, 1995.

Beiser, Frederick. *Hegel*. New York: Routledge, 2005.

Bobzien, Susanne. "Logic". In *The Cambridge Companion to the Stoics*, edited by Brad Inwood, 85–123. Cambridge: Cambridge University Press, 2003.

Burbidge, John. "Where is the Place of the Understanding?" In *Hegel on Logic and Religion: The Reasonableness of Christianity*, 29–38. Albany: State University of New York Press, 1992.

Burge, Tyler. "Concepts, Definitions and Meanings". In *Foundations of Mind: Philosophical Essays, Volume 2*, 291–306. Oxford: Clarendon Press, 2007.

Carlson, David Gray. "Introduction". In *Hegel's Theory of the Subject*, edited by David Gray Carlson xi–xix. Basingstoke: Palgrave Macmillan, 2005.

Capozzi, Mirella and Gino Roncaglia. "Logic and the Philosophy of Logic from Humanism to Kant". In *The Development of Modern Logic*, edited by Lila Haaparanta, 78–158. Oxford: Oxford University Press, 2009.

Doz, Andre. *La logique de Hegel et les problèmes traditionnels de l'ontologie*. Paris: Vrin, 1987.

Etchemendy, John. *The Concept of Logical Consequence*. Cambridge, Mass.: Harvard University Press, 1990.

Euler, L. *Letters of Euler on different subjects in natural philosophy, addressed to a German princess*: with notes and a life of Euler, translated by David Brewster. Edinburgh: Tait, 1823.

Frank, Manfred. *Auswege aus dem Deutsche Idealismus*. Frankfurt am Main: Suhrkamp, 2007.

Franz, Michael. "Einleitung". In Gottfried Ploucquet, *Logik: Herausgegeben, übersetzt und mit einer Einleitung versehen von Michael Franz*. Hildesheim: Georg Olms Verlag: 2006.

Hamann, J. G. "Metacritique of the Critique of the Purism of Reason". In *Metacritique: The Linguistic Assault on German Idealism* edited by Jere Paul Surber. Amherst, N.Y.: Humanity Books, 2001.

Harris, Henry. *Hegel's Ladder*, 2 vols. Indianapolis: Hackett, 1997.

Hegel, G. W. F. *Werke in zwanzig Bänden*, edited by Eva Moldenhauer and Karl Markus Michel. Frankfurt am Main: Suhrkamp Verlag, 1969.

Hegel, G. W. F. *Philosophy of Nature*, edited and translated by M. J. Perry, vol. 1. London: George Allen and Unwin, 1970.

Hegel, G. W. F. *Phenomenology of Spirit*, translated by A. V. Miller. Oxford: Clarendon Press, 1977.

Hegel, G. W. F. *Lectures on the History of Philosophy*, translated by E. S. Haldane and Frances H. Simson, 3 volumes. Lincoln: University of Nebraska Press, 1995.

Hegel, G. W. F. *The Science of Logic*, translated and edited by George di Giovanni. Cambridge: Cambridge University Press, 2010.

Houlgate, Stephen. *The Opening of Hegel's Logic*. West Lafayette, Ind.: Purdue University Press, 2006.

Kant, Immanuel *Lectures on Logic*, edited and translated by J. Michael Young. Cambridge: Cambridge: University Press, 1992.

Kant, Immanuel. *Critique of Pure Reason*, edited and translated by Paul Guyer and Allen W. Wood. Cambridge: Cambridge University Press, 1998.

Marciszewski, Witold and Roman Murawski. *Mechanization of Reasoning in a Historical Perspective*. Amsterdam: Editions Rodopi, 1995.

Ploucquet, Gottfried. *Logik*. Hildesheim: Georg Olms Verlag, 2006.

Russell, Bertrand. "On Denoting", *Mind*, New Series, Vol. 14, No. 56. (Oct. 1905), pp. 479–493.

Wittgenstein, Ludwig. *Tractatus Logico-Philosophicus*, translated by C. K. Ogden with an introduction by Bertrand Russell. London: Routledge and Kegan Paul, 1922.